

Mixed QCD–EW corrections to the neutral-current Drell–Yan process

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Abstract

We report on the complete computation of the mixed QCD–electroweak corrections to the neutral-current Drell–Yan process. Our calculation holds in the entire range of dilepton invariant masses. We present phenomenological results for several kinematical distributions in the case of bare muons both in the resonant region and for high invariant masses. We also consider the forward–backward asymmetry, which is a key observable to measure the weak mixing angle. We finally extend our calculation to dressed leptons and compare our results in the massless limit to those available in the literature.

1 Introduction

The Drell–Yan (DY) process [1] has played a crucial role in the historical development of the theory of the strong and electroweak (EW) interactions. It corresponds to the inclusive hadroproduction of a lepton pair through an off-shell vector boson. The production rates are large and the experimental signatures are clean, given the presence of at least one lepton with large transverse momentum in the final state, and the absence of colour flow between initial and final state.

The DY process is crucial for the precision extraction of the SM parameters, such as proton parton distribution functions (PDFs) (see e.g. Ref. [2] and references therein), the W boson mass [3–6], the weak mixing angle [7–10] and the strong coupling constant α_S (see e.g. Refs. [11, 12]).

The DY process was one of the first hadronic reactions for which radiative corrections in the strong and EW couplings α_S and α were computed. The classic calculations of the next-to-leading-order (NLO) [13] and next-to-next-to-leading-order (NNLO) [14, 15] corrections to the total cross section in QCD were followed by (fully) differential NNLO computations including the leptonic decay of the vector boson [16–20]. The complete EW corrections for W production have been computed in Refs. [21–25], and for Z production in Refs. [26–30]. Recently, the next-to-next-to-next-to-leading-order (N³LO) QCD radiative calculations for the inclusive production of a virtual photon [31, 32] and of a W boson [33] have been completed, and also computations of fiducial cross sections at this order were performed [34–37].

The mixed QCD–QED corrections to the inclusive production of an on-shell Z boson were obtained in Ref. [38] through an abelianisation procedure from the NNLO QCD results [14, 15], and later extended in Ref. [39] to the fully differential level for off-shell Z boson production and decay into a pair of neutrinos, thereby avoiding final-state radiation. A similar calculation was carried out in Ref. [40] in an on-shell approximation for the Z boson, but including the factorised NLO QCD corrections to Z production and the NLO QED corrections to the leptonic Z decay. Complete $\mathcal{O}(\alpha_S\alpha)$ computations for the production of on-shell Z and W bosons have been presented in Refs. [41–46]. Beyond the on-shell approximation, important results have been obtained in the *pole* approximation (see e.g. Ref. [47] and references therein). In this approximation the cross section around the W or Z resonance is systematically expanded so as to split the radiative corrections into well-defined, gauge-invariant contributions. Such method has been used in Refs. [48, 49] to evaluate the dominant part of the mixed QCD–EW corrections in the resonant region. The calculation in this approximation was recently completed in Ref. [50].

A first step beyond the pole approximation has been carried out in Ref. [51], where results for the $\mathcal{O}(n_F\alpha_S\alpha)$ contributions to the DY cross section were presented. In Ref. [52] some of us presented a computation of the mixed QCD–EW corrections to the charged-current process $pp \rightarrow \ell\nu_\ell + X$, where all contributions are evaluated exactly except for the finite part of the two-loop amplitude, which is evaluated in the pole approximation. Recently, the exact evaluation of the two-loop amplitude for the QCD–EW corrections to this process has been presented in Ref. [53].

The complete computation of the mixed QCD–EW corrections to the neutral-current process $pp \rightarrow \ell^+\ell^- + X$ has been reported in Ref. [54] considering massive leptons, and in Ref. [55] for massless leptons. The calculation of Ref. [54] is based on the exact two-loop amplitudes presented in Ref. [56], evaluated through the reduction to the master integrals of Ref. [57] and a semi-analytical implementation of the differential-equations method for their calculation [58–60]. The computation of Ref. [55] is based on the two-loop amplitudes presented in Ref. [61], expressed in terms of the master integrals evaluated in analytic polylogarithmic form [62, 63].

The role of the mixed NNLO QCD–EW corrections is central for a correct interpretation of high-precision DY data [64–66]. According to the factorization theorem, we can write the hadron level cross section as a convolution of the proton PDFs with a partonic cross section, and both factors feature QCD and EW radiative effects. Initial-state QED and mixed QCD–QED collinear singularities can be factorized from the partonic cross section and reabsorbed in the definition of the physical proton PDF [67, 68]. The coupled DGLAP evolution, with QCD and QED splitting kernels, yields a non-trivial interplay which goes beyond the simple single-emission probability, with mixed higher-order effects automatically included [69–71]. The presence of a photon density in the proton opens the possibility of new partonic scattering channels, necessary to complement the cross sections initiated by quarks and gluons. The relative contribution of the various channels changes, as a function of the invariant mass of the final state, and only an NNLO QCD–EW calculation can consistently account for their correct balance [72–74]. The availability of an exact NNLO QCD–EW calculation throughout the whole invariant-mass range, including both the Z resonance and the large invariant-mass tail, is important to predict the shape of the kinematical distributions, and, in turn, to establish in a statistically significant way any discrepancy from the data. In a distinct and complementary perspective, the increased precision of the partonic results may help putting tighter constraints on the proton parameterization at large partonic x [75]. Moreover, the knowledge of the NNLO QCD–EW corrections potentially allows us to improve over the approximations used in shower Monte Carlo programs (see e.g. Refs. [76, 77], Sec. IV.1 of Ref. [78], and the benchmarking study of Ref. [79]), which include only partial subsets of factorisable mixed QCD–EW corrections, and to reduce the remaining theoretical uncertainties.

In this paper we follow up on the earlier results presented in Ref. [54] and present a detailed study on the impact of mixed QCD–EW corrections on the neutral-current DY process at LHC energies. We start by considering bare muons and examine the mixed QCD–EW corrections in both the resonant region and in the region of high invariant masses. We then analyse the impact of radiative corrections on the forward–backward asymmetry. We finally consider the case of dressed leptons. Here we show that our calculation can be extended to the massless limit, and we present a comparison of our results with those of the massless calculation of Ref. [55].

The paper is organised as follows. In Sec. 2 we provide some details of our calculation. In Sec. 3 we present our phenomenological results, starting from the resonant region (Sec. 3.1) and then moving to the region of high invariant masses (Sec. 3.2). We further present results for the forward–backward asymmetry (Sec. 3.3), and finally consider the massless limit by presenting a comparison with the results of Ref. [55] (Sec. 4). Our results are summarised in Sec. 5.

2 Computational details

We consider the inclusive production of a pair of massive muons in proton–proton collisions,

$$pp \rightarrow \mu^+ \mu^- + X. \quad (1)$$

The theoretical predictions for this process can be obtained as a convolution of the parton distribution functions for the incoming protons with the hard-scattering partonic cross sections. When QCD and EW radiative corrections are considered, the initial partons include quarks, anti-quarks, gluons and photons.

The differential cross section for the process in Eq. (1) can be written as

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)}, \quad (2)$$

where $d\sigma^{(0,0)} \equiv d\sigma_{\text{LO}}$ is the Born level contribution and $d\sigma^{(m,n)}$ the $\mathcal{O}(\alpha_S^m \alpha^n)$ correction. The mixed QCD–EW corrections correspond to the term $m = n = 1$ in this expansion and include double-real, real–virtual and purely virtual contributions. The corresponding tree-level and one-loop scattering amplitudes are computed with OPENLOOPS [80–82] and RECOLA [83–85], the results being in complete agreement¹. The two-loop amplitude [56] is computed exploiting the reduction of the scalar integrals appearing in the corresponding Feynman diagrams to master integrals [57, 62] using KIRA [86]; their subsequent computation uses the semi-analytic approach to the differential-equations method [58] as implemented in the Mathematica package SeaSyde [60]. The masses of the vector bosons are renormalized in the complex-mass scheme [87] and consistently kept complex-valued at each step of the computation. Even when all the amplitudes have been computed, the completion of the calculation remains a formidable task. Indeed, double-real, real–virtual and purely virtual contributions are separately infrared divergent, and a method to handle and cancel infrared singularities has to be worked out. In this calculation we use a formulation of the q_T subtraction formalism [88] derived from the NNLO QCD computation of heavy-quark pair production [89–91] through an appropriate abelianisation procedure [38, 92]. According to the q_T subtraction formalism [88], $d\sigma^{(m,n)}$ can be evaluated as

$$d\sigma^{(m,n)} = \mathcal{H}^{(m,n)} \otimes d\sigma_{\text{LO}} + \left[d\sigma_{\text{R}}^{(m,n)} - d\sigma_{\text{CT}}^{(m,n)} \right]. \quad (3)$$

The first term in Eq. (3) is obtained through a convolution (denoted by the symbol \otimes) of the perturbatively computable function $\mathcal{H}^{(m,n)}$ and the LO cross section $d\sigma_{\text{LO}}$, with respect to the longitudinal-momentum fractions of the colliding partons. The second term is the *real* contribution $d\sigma_{\text{R}}^{(m,n)}$, where the charged leptons are accompanied by additional QCD and/or QED radiation that produces a recoil with finite transverse momentum q_T . For $m + n = 2$ such contribution can be evaluated by using the dipole subtraction formalism [93–100]. In the limit $q_T \rightarrow 0$ the real contribution $d\sigma_{\text{R}}^{(m,n)}$ is divergent, since the recoiling radiation becomes soft

¹To be precise, the results obtained with OPENLOOPS and RECOLA coincide pointwise in phase space when adopting the G_μ or $\alpha(M_Z)$ EW renormalization schemes. They differ in the case of the $\alpha(0)$ scheme because of the different treatment of the light-fermion contributions to the photon vacuum polarization.

and/or collinear to the initial-state partons. Such divergence is cancelled by the counterterm $d\sigma_{\text{CT}}^{(m,n)}$, which eventually makes the cross section in Eq. (3) finite.²

The required phase space generation and integration is carried out within the MATRIX framework [101]. The core of MATRIX is the Monte Carlo program MUNICH³, which contains a fully automated implementation of the dipole subtraction method for massless and massive partons at NLO QCD [93–95] and NLO EW [96–100]. The q_T subtraction method has been applied to several NNLO QCD computations for the production of colourless final-state systems (see Ref. [101] and references therein), and to heavy-quark pair production [89–91] and related processes [102–105], which correspond to the case $m = 2$, $n = 0$. The method has also been used in Ref. [92] to study NLO EW corrections for the DY process, which represents the case $m = 0$, $n = 1$. In the last few years some of us have applied this method to the computation of mixed QCD–EW corrections for the charged-current DY process [52], while first results on the neutral-current DY process were presented in Ref. [54]. The structure of the coefficients $\mathcal{H}^{(1,1)}$ and $d\sigma_{\text{CT}}^{(1,1)}$ can be derived from those controlling the NNLO QCD computation of heavy-quark pair production. The initial-state soft/collinear and purely collinear contributions were already presented in Ref. [39]. The fact that the final state is colour neutral implies that final-state radiation is of pure QED origin. Therefore, the purely soft contributions have a simpler structure than the corresponding contributions entering the NNLO QCD computation of heavy-quark pair production [106].

3 Phenomenological results

Unless stated otherwise, we work in the G_μ scheme with $G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}$ and set the *on-shell* values of masses and widths to $m_{W,\text{OS}} = 80.385 \text{ GeV}$, $m_{Z,\text{OS}} = 91.1876 \text{ GeV}$, $\Gamma_{W,\text{OS}} = 2.085 \text{ GeV}$, $\Gamma_{Z,\text{OS}} = 2.4952 \text{ GeV}$. Those values are translated to the corresponding *pole* values $m_V = m_{V,\text{OS}}/\sqrt{1 + \Gamma_{V,\text{OS}}^2/m_{V,\text{OS}}^2}$ and $\Gamma_V = \Gamma_{V,\text{OS}}/\sqrt{1 + \Gamma_{V,\text{OS}}^2/m_{V,\text{OS}}^2}$, $V = W, Z$, from which $\alpha = \sqrt{2} G_F m_W^2 (1 - m_W^2/m_Z^2)/\pi$ is derived, and we use the complex-mass scheme [87] throughout. The muon mass is fixed to $m_\mu = 105.658369 \text{ MeV}$, and the pole masses of the top quark and the Higgs boson to $m_t = 173.07 \text{ GeV}$ and $m_H = 125.9 \text{ GeV}$, respectively. The CKM matrix is taken to be diagonal. We work with $n_f = 5$ massless quark flavours and retain the exact top-mass dependence in all virtual and real–virtual amplitudes associated with bottom-induced processes, except for the two-loop virtual corrections, where we neglect top-mass effects. Given the smallness of the bottom-quark density, we estimate the corresponding error to be at the percent level of the computed correction. We use the NNPDF31_nnlo_as_0118_luxqed set of parton distributions [107], which is based on the LUXqed methodology [73] for the determination of the photon density. If not stated otherwise, the central values of the renormalisation and factorisation scales are fixed to $\mu_R = \mu_F = m_Z$ (and the corresponding value of α_S is set), while theoretical uncertainties are estimated by the customary 7-point scale variation, i.e. by varying

²More precisely, the square bracket in Eq. (3) is evaluated with a cut $q_T/m_{\mu\mu} > r_{\text{cut}}$ ($m_{\mu\mu}$ being the invariant mass of the dimuon system), and then an extrapolation for $r_{\text{cut}} \rightarrow 0$ is carried out (see Ref. [101] for more details).

³MUNICH, which is the abbreviation of “MULTI-channel Integrator at Swiss (CH) precision”, is an automated parton-level NLO generator by S. Kallweit.

μ_R and μ_F by a factor of two around their central values with the constraint $1/2 < \mu_R/\mu_F < 2$.

Our results for the mixed QCD–EW corrections will be compared with those from an approach in which QCD and EW corrections are assumed to completely factorise. Such approximation can be defined as follows: for each bin of a distribution $d\sigma/dX$, the QCD correction, $d\sigma^{(1,0)}/dX$, and the EW correction restricted to the $q\bar{q}$ channel, $d\sigma_{q\bar{q}}^{(0,1)}/dX$, are computed, and the factorised $\mathcal{O}(\alpha_S\alpha)$ correction is calculated as

$$\frac{d\sigma_{\text{fact}}^{(1,1)}}{dX} = \left(\frac{d\sigma^{(1,0)}}{dX}\right) \times \left(\frac{d\sigma_{q\bar{q}}^{(0,1)}}{dX}\right) \times \left(\frac{d\sigma_{\text{LO}}}{dX}\right)^{-1}. \quad (4)$$

This approximation is justified if the dominant sources of QCD and EW corrections factorise with respect to the hard gauge-boson production subprocess (see the discussion in Ref. [52]).

3.1 Resonant region

We first study the impact of the mixed corrections on the bulk of the DY cross section around the Z peak. We consider proton–proton collisions at $\sqrt{s} = 13.6$ TeV and a fiducial volume defined by staggered cuts⁴ on the transverse momenta of the positively/negatively charged leptons,

$$p_{T,\mu^+} > 27 \text{ GeV}, \quad p_{T,\mu^-} > 25 \text{ GeV}, \quad |y_\mu| < 2.5, \quad (5)$$

while the invariant mass of the dimuon pair $m_{\mu\mu}$ fulfils

$$66 \text{ GeV} < m_{\mu\mu} < 116 \text{ GeV}. \quad (6)$$

In Table 1 we show our predictions for the fiducial cross section with an increasing radiative content. We start our discussion from the pure QCD predictions, labelled LO, NLO_{QCD} and NNLO_{QCD}. The NLO QCD corrections increase the fiducial cross section by about 7% with respect to the LO, their perturbative uncertainty, estimated through scale variations, being about 4%. The NNLO QCD corrections are negative, and amount to about -2% , while the associated uncertainties are reduced to below the percent level. N³LO corrections with this setup are not available, but their impact is likely to exceed the NNLO scale uncertainties [35].

Predictions including NLO EW effects are shown in the fourth and fifth rows of Table 1, and are labelled NLO_{EW} and NNLO_{QCD+NLO_{EW}}, respectively, while our best prediction, which includes mixed QCD–EW corrections as well, is labelled NNLO_{QCD+MIX+NLO_{EW}}. We see that NLO EW corrections are negative, about -4% , and have a larger impact than what might be expected by a naive coupling counting. Indeed, they are similar in size to the NLO QCD effects and, being negative, largely compensate the latter. We observe that the scale dependence of the NLO_{EW} prediction is identical to that at LO, as might have been expected, given that the relevant amplitudes do not involve the QCD coupling α_S . The newly computed mixed QCD–EW corrections are rather small for this particular setup, being at the few per mille level. They

⁴This combination of cuts is known to restore the quadratic dependence of the acceptance at small transverse momentum of the dilepton pair, see e.g. Refs [101, 108].

slightly reduce the fiducial cross sections. The small size of the mixed QCD–EW corrections is consistent with the fact that they do not reduce the scale dependence.

	σ [pb]	$\sigma^{(i,j)}$ [pb]	$\sigma^{(i,j)}/\sigma_{\text{LO}}$
LO	735.80(2) $^{+12.7\%}_{-13.6\%}$	–	–
NLO _{QCD}	790.23(5) $^{+2.7\%}_{-4.4\%}$	$\sigma^{(1,0)} = +54.42(5)$	+7.4%
NNLO _{QCD}	773.2(8) $^{+0.7\%}_{-0.7\%}$	$\sigma^{(2,0)} = -17.0(8)$	-2.3%
NLO _{EW}	704.15(2) $^{+12.7\%}_{-13.6\%}$	$\sigma^{(0,1)} = -31.65(3)$	-4.3%
NNLO _{QCD} +NLO _{EW}	741.5(8) $^{+0.3\%}_{-0.7\%}$	–	–
NNLO _{QCD} +MIX+NLO _{EW}	739.1(1.1) $^{+0.7\%}_{-0.7\%}$	$\sigma^{(1,1)} = -2.4(7)$	-0.33%

Table 1: Fiducial cross section at the different perturbative orders and the corresponding corrections $\sigma^{(i,j)}$ as defined in Eq. (2).

We now turn to differential distributions. In Fig. 1 (left) we show our results for the rapidity distribution $y_{\mu\mu}$ of the dimuon pair. The strongest shape effect can be observed at NLO QCD and to a lesser extent at NNLO QCD. The inclusion of the mixed QCD–EW corrections further distorts the shape of the distribution up to 1% in the region $|y_{\mu\mu}| > 1.5$, and could be potentially relevant for a determination of the proton PDFs with 1% precision [109].

In Fig. 1 (right), we consider the invariant-mass distribution of the dimuon system. QCD radiative corrections are generally mild for this observable, while the emission of QED radiation off the final-state lepton pair has a significant impact if the invariant mass is reconstructed from bare leptons, as we do here. The inclusion of the NLO EW corrections indeed leads to a shift of events, giving rise to the formation of the characteristic radiative tail in the region on the left of the peak.⁵ Purely weak NLO corrections modify the strength of the Z -boson couplings to fermions and in turn, at LO, the squared matrix element of the Z -exchange diagram and the γ – Z interference. They affect the region around the Z resonance, with positive corrections reaching up to a few percent below the resonance and moderate negative corrections above the resonance [50].

Focusing on the impact of the mixed QCD–EW corrections in the middle plot, we observe sizeable effects and a non-trivial shape distortion. The correction is vanishing around the peak, where it reaches the maximum slope, while it is rather flat, positive and around 4% in the region below the peak and rather flat, negative and around -2% in the region above. We observe that the prediction obtained with the factorised ansatz (see Eq. (4)) works well in the latter region, while it fails to describe the exact result below the Z resonance, because of a non-trivial kinematical interplay between the production of the gauge boson and its subsequent decay into leptons, as it has been first pointed out in Ref. [49]. Furthermore, we notice that the inclusion

⁵The effect is also present but reduced by approximately a factor of two when leptons and photons are recombined [28].

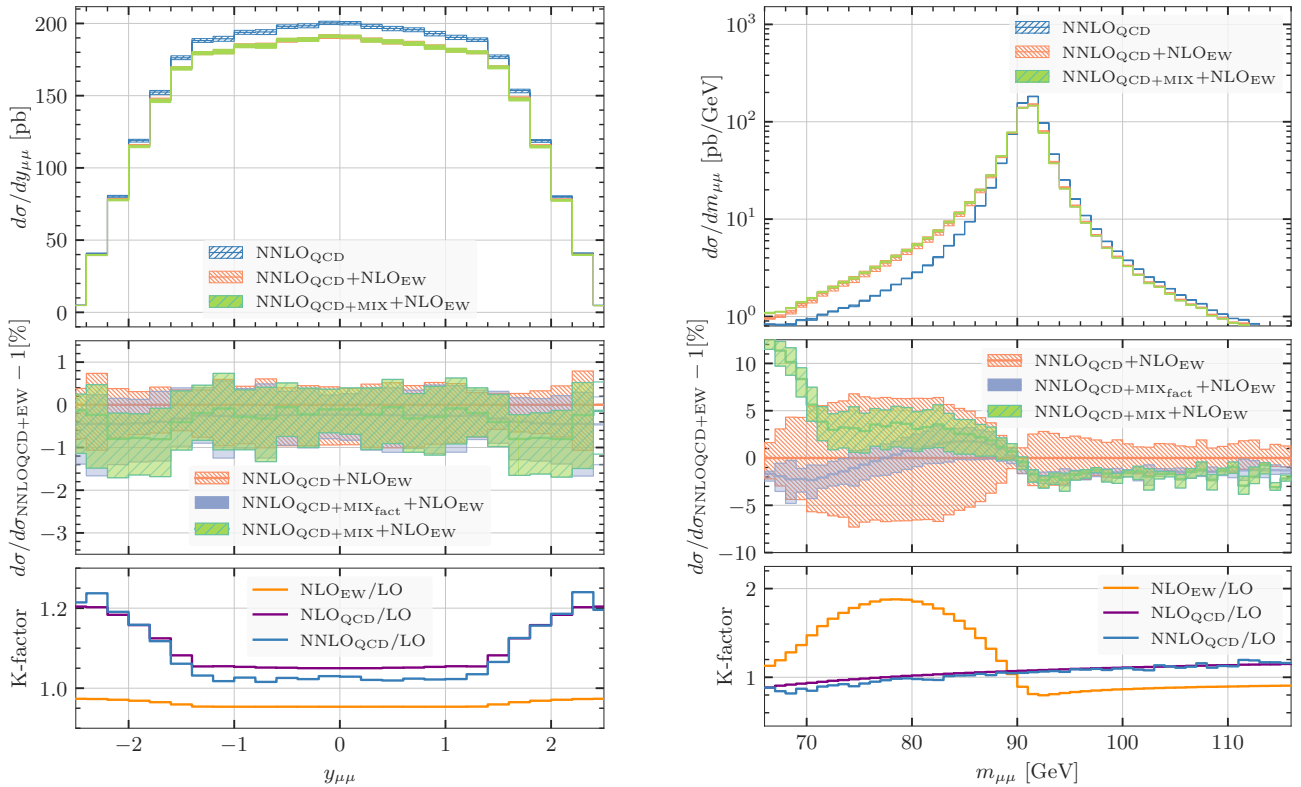


Figure 1: Predictions for the rapidity distribution (left) and the invariant-mass distribution (right) of the final-state muon pair, in the setup defined in Section 3.1. NLO EW and mixed QCD–EW corrections are additively included on top of the NNLO QCD prediction. The relative effects of the mixed corrections on top of the additive combination NNLO QCD + NLO EW are shown in the middle panel, comparing the exact mixed corrections to the approximate factorised ansatz defined in the text. NLO EW as well as NLO and NNLO QCD K -factors are displayed in the bottom panel.

of the mixed corrections leads to a significant reduction of the uncertainty band.

In Fig. 2 we show the rapidity distribution of the μ^+ (left) and the $\cos \theta^*$ distribution (right). The Collins–Soper (CS) angle θ^* [110] is defined in terms of LAB frame variables as

$$\cos \theta^* = \frac{y_{\mu\mu}}{|y_{\mu\mu}|} \frac{2(p_{\mu^-}^+ p_{\mu^+}^- - p_{\mu^-}^- p_{\mu^+}^+)}{m_{\mu\mu} \sqrt{m_{\mu\mu}^2 + p_{T,\mu\mu}^2}}, \quad (7)$$

with $p^\pm = (p^0 \pm p^3)/\sqrt{2}$. The factor $y_{\mu\mu}/|y_{\mu\mu}|$ takes into account the fact that in hadron collisions the quark and anti-quark directions are not known for each collision. However, on average the quark carries more energy than the anti-quark as the latter must originate from the sea. Then, the produced dilepton system is, on average, boosted in the direction of the valence quark. Since the Z boson rapidity is correlated with the direction of the valence quark, it offers a sensible reference to define the scattering angle.

The radiative corrections to the rapidity distribution of the anti-muon (Fig. 2 (left)) have a rather uniform impact over the considered rapidity range. In the case of the $\cos \theta^*$ distribution,

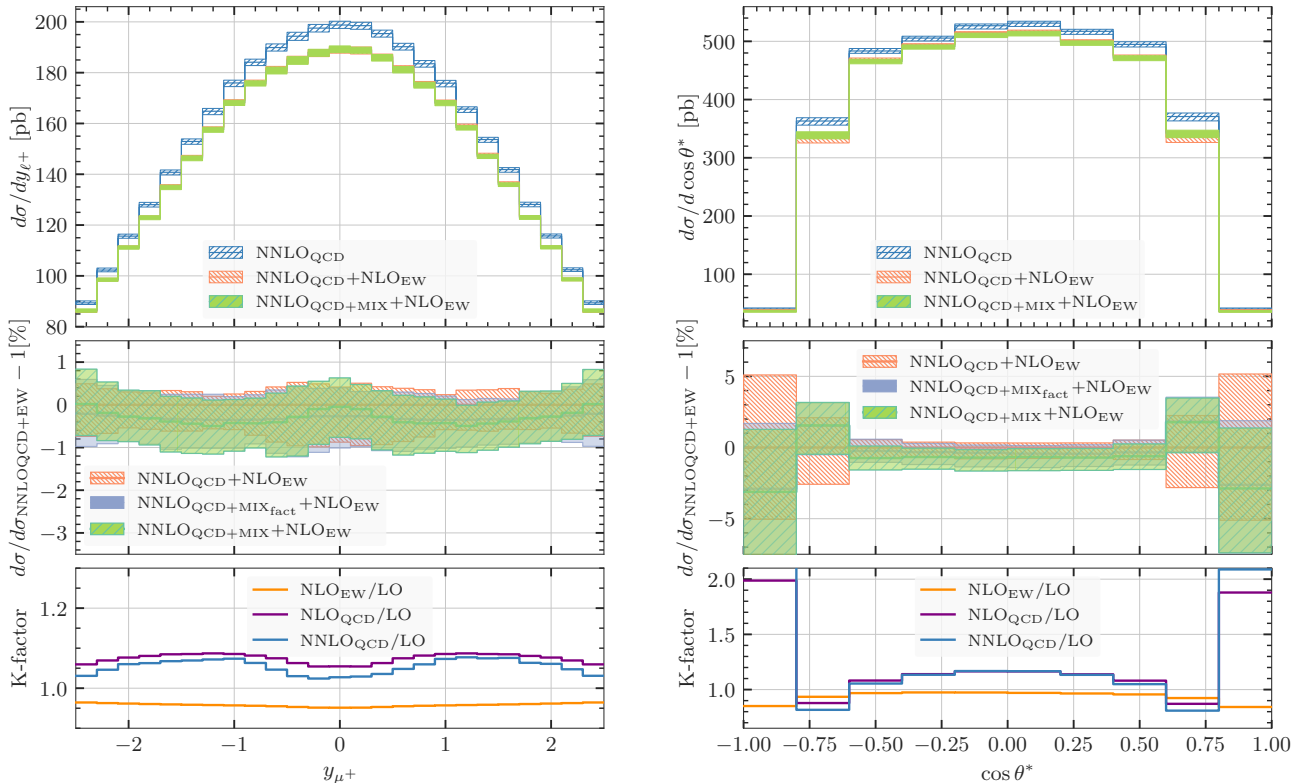


Figure 2: Same as Fig. 1 for the rapidity distribution of the anti-muon (left) and the CS angle $\cos \theta^*$ (right).

radiative corrections distort the distribution in different ways. NLO QCD corrections are larger at large $|\cos \theta^*|$, which is caused by the qg channel opening up at NLO, dominated by configurations where a valence quark at relatively large momentum fraction x and a gluon at small x enter the hard process. By contrast, EW corrections distort the distribution in the opposite way, their impact being slightly larger and negative at large values of $|\cos \theta^*|$. The impact of the mixed QCD–EW corrections is generally below the percent level, reaching the few percent level in the region of large $|\cos \theta^*|$.

3.2 High invariant-mass region

Besides the resonant region, percent-level precision can be envisaged also for data taken in the region of high invariant masses. This requires a careful assessment of the impact of EW radiative corrections and their interplay with QCD effects. Following Ref. [111], we consider proton–proton collisions at $\sqrt{s} = 13$ TeV and impose the following cuts on the final-state leptons:

$$p_{T,\mu^\pm} > 53 \text{ GeV}, \quad |y_\mu| < 2.4, \quad m_{\mu\mu} > 150 \text{ GeV}. \quad (8)$$

For this setup, a dynamical scale is a more appropriate choice. We use as central values of the renormalisation and factorisation scales the invariant mass of the dimuon system $m_{\mu\mu}$.

In Fig. 3 we show the invariant-mass distribution up to $m_{\mu\mu} = 2$ TeV. Excluding the first bin

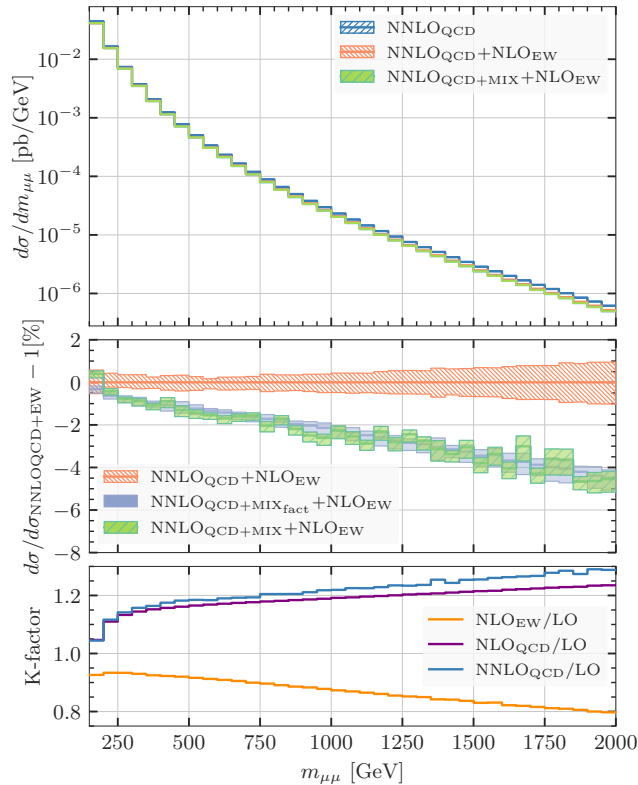


Figure 3: Dimuon invariant-mass distribution, in the acceptance setup defined in Section 3.2. The structure of the panels and the colour codes are the same as in Fig. 1.

close to the edge at $m_{\mu\mu} = 150$ GeV, imposed by the selection requirements on the leptons, the effect of the mixed corrections is negative and increasing at higher invariant masses, reaching -5% at $m_{\mu\mu} = 2$ TeV. Such values are comparable with or even larger than the statistical error expected at the end of the High-Luminosity (HL) phase of the LHC, with a total collected luminosity of 3 ab^{-1} . The inclusion of these corrections should help reducing the theoretical uncertainty on the partonic cross section of the DY process and, in turn, help constraining the proton PDFs, at large partonic x , in a global fit of collider data.⁶ The precise knowledge of the higher-order corrections to this distribution is the mandatory starting point for an analysis which aims at the determination of the running of the $\overline{\text{MS}}$ weak mixing angle [113].

We observe that the pattern of the mixed QCD–EW corrections is well approximated by the factorised ansatz (see Eq. (4)). This implies that QCD and EW effects largely factorise in this region of high invariant masses, as also observed in Ref. [55].

In Fig. 4, we show the invariant-mass distributions in the backward and forward regions up to $m_{\mu\mu} = 6$ TeV. We see that the impact of mixed corrections increases as $m_{\mu\mu}$ increases. The effect is slightly more pronounced in the forward region than in the backward region, following an analogous effect in the NLO EW corrections. As observed in Fig. 3 the factorised ansatz works rather well.

⁶The possibility that a PDF fit at large invariant masses inadvertently reabsorbs a New Physics signal has been discussed in Ref. [112].

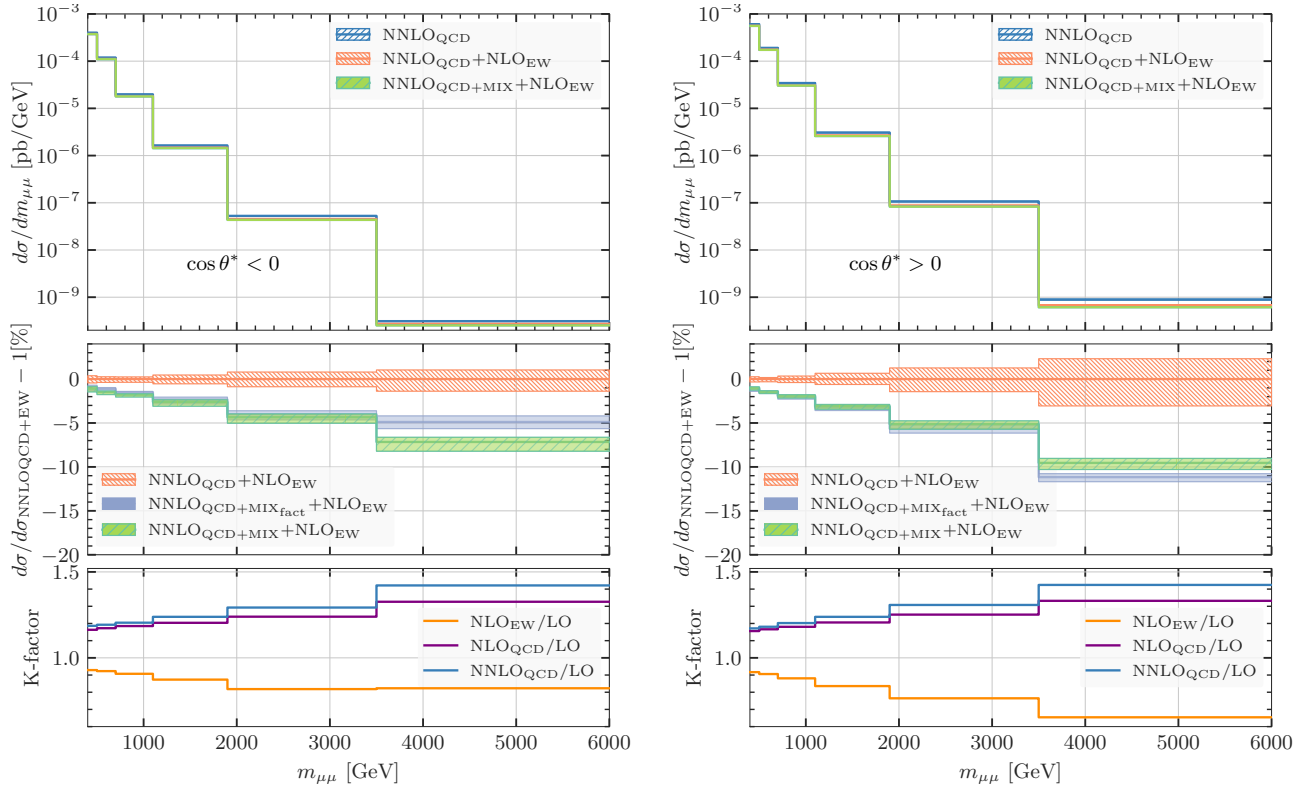


Figure 4: Dimuon invariant-mass distribution, in the acceptance setup defined in Section 3.2, in the CMS bins [111], in the backward (left) and forward (right) regions. The structure of the panels and the colour codes are the same as in Fig. 1.

3.3 Forward–Backward asymmetry

The Forward–Backward (FB) asymmetry $A_{FB}(m_{\ell\ell})$ is one of the main observables for the determination of the leptonic effective weak mixing angle $\sin^2\theta_{\text{eff}}^\ell$. At the LHC, $A_{FB}(m_{\ell\ell})$ is defined as the difference between the cross sections in the forward and backward directions normalised to the total cross section, differentially with respect to the invariant mass $m_{\ell\ell}$ of the dilepton system [26, 114],

$$A_{FB}(m_{\ell\ell}) = \frac{F(m_{\ell\ell}) - B(m_{\ell\ell})}{F(m_{\ell\ell}) + B(m_{\ell\ell})}. \quad (9)$$

The forward and backward cross sections are given by

$$F(m_{\ell\ell}) = \int_0^1 d\cos\theta^* \frac{d\sigma}{d\cos\theta^* dm_{\ell\ell}}(m_{\ell\ell}), \quad B(m_{\ell\ell}) = \int_{-1}^0 d\cos\theta^* \frac{d\sigma}{d\cos\theta^* dm_{\ell\ell}}(m_{\ell\ell}), \quad (10)$$

where $\cos\theta^*$ is defined in Eq. (7).

The FB asymmetry is defined from the identification of the scattering angle of the negatively charged outgoing lepton, with respect to the direction of the incoming particle, in our case at LO a quark. In proton–proton collisions, we cannot prepare the partonic initial state or, in other words, we are not aware of the direction of the incoming quark: each final state receives, at LO,

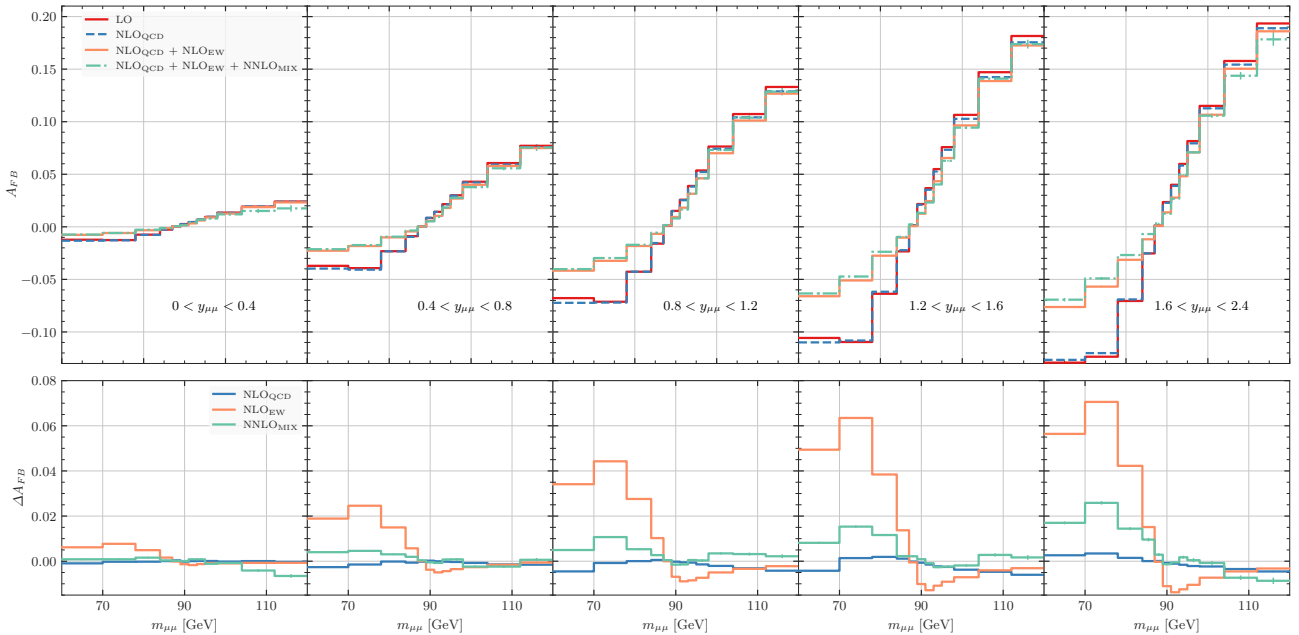


Figure 5: Theoretical predictions for the FB asymmetry for different slices of rapidity of the dimuon final state at different perturbative orders. Shifts with respect to the LO prediction induced by separate sets of corrections are reported in the bottom panels.

contributions from both quark–antiquark and antiquark–quark annihilation, listing the partons involved in the hard scattering process from the first and the second hadron, respectively. If the partonic centre-of-mass system is at rest in the laboratory frame, $y_{\ell\ell} = 0$, the invariance of the system under interchange of the incoming hadrons exactly cancels out all the parity-violating contributions to $A_{FB}(m_{\ell\ell})$. At non-vanishing $y_{\ell\ell}$ values, the quark–antiquark subprocess (with the incoming quark oriented along the rapidity direction) prevails over the antiquark–quark process, because of the larger weight of the quark PDF with its valence content than that of the antiquark PDF; the unbalance avoids the cancellation of the parity-violating effects, and we observe a non-vanishing $A_{FB}(m_{\ell\ell})$. The slope of the asymmetry, in the Z resonance region, is steeper for larger $y_{\ell\ell}$ values, as illustrated by the different panels in Fig. 5.

The experiments run at LEP/SLC [115] have measured the leptonic FB asymmetry at the Z resonance with a precision of about 6%, which in turn allowed a determination of $\sin^2 \theta_{\text{eff}}^\ell$ with a relative error of about $7 \cdot 10^{-4}$. Measurements of the FB asymmetry have been performed by the CMS experiment at 8 TeV [116], by the ATLAS experiment at 7 TeV [117] and by the LHCb experiment exploiting data at 7 and 8 TeV [8]. More recently, the CMS experiment has measured $A_{FB}(m_{\ell\ell})$ and extracted $\sin^2 \theta_{\text{eff}}^\ell$ from data collected during Run 2 at 13 TeV [10]. These measurements have already reached a precision better than the percent level.

Given the foreseen increase in statistics in the HL-LHC phase, it is important to investigate the impact of higher-order QCD, EW, and mixed QCD–EW radiative corrections in the theory predictions of $A_{FB}(m_{\ell\ell})$. These effects have been recently studied in the pole approximation [50] in proton–proton collisions at 13 TeV. In the following, we present a study on the impact of the exact mixed QCD–EW corrections to the FB asymmetry, for dimuon production in proton–proton collisions at 8 TeV. We adopt the setup used by the CMS collaboration for the extraction

of $\sin^2 \theta_{\text{eff}}^\ell$ [116]:

$$p_{T,1} > 25 \text{ GeV}, \quad p_{T,2} > 15 \text{ GeV}, \quad |y_\mu| < 2.4, \quad 60 < m_{\mu\mu} < 120 \text{ GeV}, \quad (11)$$

where $p_{T,1}$ ($p_{T,2}$) is the transverse momentum of the hardest (second hardest) lepton. We consider five intervals in the absolute rapidity of the dimuon final state: $0 < |y_{\mu\mu}| < 0.4$, $0.4 < |y_{\mu\mu}| < 0.8$, $0.8 < |y_{\mu\mu}| < 1.2$, $1.2 < |y_{\mu\mu}| < 1.6$ and $1.6 < |y_{\mu\mu}| < 2.4$. In order to better quantify the impact of the higher-order corrections, we consider the shifts with respect to the LO prediction for the FB asymmetry,

$$\Delta A_{FB}^X(m_{\ell\ell}) = A_{FB}^X(m_{\ell\ell}) - A_{FB}^{\text{LO}}(m_{\ell\ell}), \quad (12)$$

where $X = \text{NLO}_{\text{QCD}}$, NLO_{EW} or NNLO_{MIX} ⁷. In Fig. 5, we show results for $A_{FB}(m_{\ell\ell})$ with an increasing radiative content. The shifts compared to the LO asymmetry are shown in the bottom panels of Fig. 5.

We first observe that the NLO QCD corrections have a tiny impact throughout the whole invariant-mass range: the strong interaction is invariant under parity, and QCD corrections largely cancel in the definition of $A_{FB}(m_{\ell\ell})$. The NLO EW corrections have a large impact, through two distinct mechanisms: the large parity-even QED correction contributes mostly due to final-state radiation, which is particularly important below the Z resonance; the purely weak correction contributes instead to the parity-violating contribution. These two effects combined yield the largest shift of $A_{FB}(m_{\ell\ell})$. The mixed QCD–EW corrections inherit both features from the NLO EW terms, with increasing impact at larger $y_{\mu\mu}$ values; they are larger in size than the purely QCD corrections.

The effective weak mixing angle $\sin^2 \theta_{\text{eff}}^\ell$ can be determined from the study of $A_{FB}(m_{\ell\ell})$ [118]. In the Z -resonance region $A_{FB}(m_{\ell\ell})$ is generated by the product of vector and axial-vector couplings of the Z boson to fermions, in the squared Z -exchange diagrams, yielding in turn a sensitivity to the value of the weak mixing angle, present in the vector coupling. Away from the resonance, the squared Z -exchange diagrams are kinematically suppressed, $A_{FB}(m_{\ell\ell})$ is larger than at the resonance and driven by the product of the photon vector coupling with the axial coupling of the Z boson. This second coupling combination has no dependence on the weak mixing angle.

The $\sin^2 \theta_{\text{eff}}^\ell$ determination cannot be based on the analysis of one single bin of the $A_{FB}(m_{\ell\ell})$ invariant-mass distribution, i.e. the Z -resonance bin, because the result would be swamped by PDF uncertainties. The latter can be constrained [10], adopting a template fit procedure, which typically requires the analysis of $A_{FB}(m_{\ell\ell})$ in the mass window [70, 110] GeV. It is thus necessary to have control over the radiative corrections in this very region.

In order to illustrate the sensitivity of $A_{FB}(m_{\ell\ell})$ to the weak mixing angle, in Fig. 6 we study the variation of the asymmetry induced by a change of the W -boson mass, one of our input parameters, of 10 MeV, which corresponds to a shift of the weak mixing angle by $1.9 \cdot 10^{-4}$. Comparing Fig. 5 and Fig. 6, we clearly see that the impact of the mixed QCD–EW corrections

⁷Here NNLO_{MIX} includes only the mixed QCD–EW corrections on top of the LO prediction.

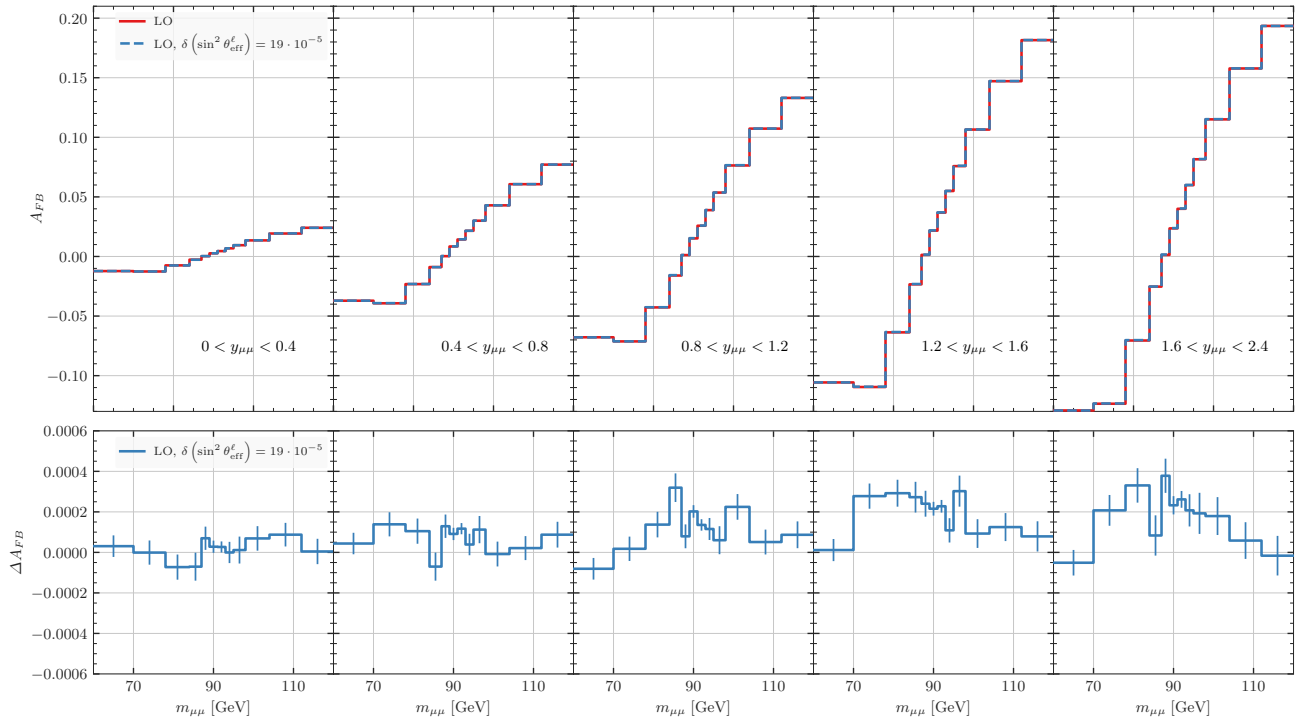


Figure 6: The variation of $A_{FB}(m_{\ell\ell})$ for a change of the weak mixing angle value by $19 \cdot 10^{-5}$.

is large. Part of the effect is known to be due to QED final-state radiation [50], which is typically accounted for in the experimental analyses using Monte Carlo parton showers. Nonetheless, when comparing Fig. 5 and Fig. 6, it is evident that a thorough evaluation of the computed corrections will be essential for accurate future determinations of the effective electroweak mixing angle $\sin^2 \theta_{\text{eff}}^\ell$.

4 Comparison with massless calculation

In this Section we present a comparison of our results to those of Ref. [55]. To this purpose, the EW scheme and input parameters are tuned to those of Ref. [55], as far as possible. In Ref. [55] the selection cuts are

$$m_{\ell\ell} > 200 \text{ GeV}, \quad p_{T,\ell^\pm} > 30 \text{ GeV}, \quad \sqrt{p_{T,\ell^+} p_{T,\ell^-}} > 35 \text{ GeV}, \quad |y_{\ell^\pm}| < 2.5, \quad (13)$$

and leptons are treated as massless.

It is well known that fiducial cuts applied to *bare* massless leptons are, in general, not collinear safe with respect to the emission of a collinear photon off a final-state lepton. The definition of physical observables requires therefore the introduction of a recombination procedure of photons to a close-by lepton. This prevented the authors of Ref. [55] to directly compare their results, valid for *dressed* or *calo* leptons, to the results of Ref. [54], obtained for the case of bare massive muons.

Conversely, our framework, which retains the dependence on the mass of the final-state

leptons, allows us to perform calculations for both bare and dressed leptons. This can be easily understood if we take seriously the fact that the mass of the lepton acts as the regulator of the final-state collinear singularity associated with the emission of a photon off a lepton. Indeed, the resulting dead-cone effect leads to a smooth suppression of the collinear photon radiation at the parton level. In combination with the cut on the transverse momentum of the dilepton system, again considered at the parton/bare level, this makes the real-emission cross section finite. We can then apply the recombination procedure to each parton level event passing the $q_T/m_{\ell\ell} > r_{\text{cut}}$ requirement. This is perfectly analogous to what happens for jet cross sections computed with a local subtraction method. We also stress that the mass of the lepton should not be regarded as an additional cut-off parameter in our formalism (besides the slicing parameter r_{cut}). Our subtraction formula in Eq. (3) indeed retains the exact dependence on the mass of the lepton, without introducing further approximations.

To summarise, we can compare with the results of Ref. [55] for dressed leptons within our framework by repeating the calculation for decreasing lepton masses and eventually taking the limit $m_\ell \rightarrow 0$. We stress again that the cut on the transverse momentum must be applied at the parton level, *before* any recombination takes place. In practice, we can perform the calculation at a sufficiently small value of the lepton mass, which can be even larger than the physical mass of the considered leptons, as long as power corrections can be safely neglected. For the sake of the comparison, we reproduce the results for dressed electrons of Ref. [55] using as reference lepton mass $m_\ell = m_\mu = 105.658369$ MeV.

The above procedure is consistent with the fact that we are performing an on-shell renormalisation of the electric charge. Instead, one can consider a running coupling constant as, for example, in the $\overline{\text{MS}}$ scheme with only massless fermions (quarks and leptons) actively running in the beta function, while the wave functions of the external massive leptons are renormalised on-shell (decoupling scheme). In this situation, it is well known that an extra correction contribution, proportional to $\ln \mu_R/m_\ell$, must be added in passing from the massive to the massless calculation in order to compensate for the different number of active flavours in the beta function [119].

There is, however, a caveat related to LO photon-induced processes, namely the $\gamma\gamma \rightarrow \ell^+\ell^-$ channel. In this case, the subtraction of the photonic vacuum polarisation insertions is usually dealt with relying on dimensional regularisation for the light flavours, whereas, for massive fermions, the mass is used as the physical regulator. This is consistent with the reabsorption of initial-state collinear singularities into the renormalisation of the parton distribution functions when applying collinear factorisation, with the light flavours treated as active in the $\overline{\text{MS}}$ scheme, whereas the massive flavours are only generated radiatively from scales above the pair production threshold. Therefore, in this case, the procedure stated above must be supplemented by adding a contribution which compensates for the different treatment in the collinear factorisation. In other words, the photon distribution receives an $\mathcal{O}(\alpha)$ correction due to the change of scheme [120], and the cross section in the $\gamma\gamma$ channel is modified as

$$d\sigma_{\gamma\gamma, m_\ell=0}^{(0,1)} = d\sigma_{\gamma\gamma, m_\ell}^{(0,1)} - \alpha \frac{2e_\ell^2}{3\pi} \ln \frac{m_\ell^2}{\mu_F^2} d\sigma_{\gamma\gamma, m_\ell}^{(0,0)}, \quad (14)$$

σ [pb]	σ_{LO}	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	1561.52(5)	340.3(3)	-49.77(5)	44.6(4)	-17.2(7)
qg	—	0.0601(3)	—	-32.7(2)	2.09(9)
$q\gamma$	—	—	-0.30(2)	—	-0.230(3)
$g\gamma$	—	—	—	—	0.2648(17)
gg	—	—	—	2.02(6)	—
$\gamma\gamma$	59.645(6)	—	3.174(9)	—	—

Table 2: The different perturbative contributions to the fiducial cross section in the various partonic channels, in the setup of Ref. [55].

which is obtained through the abelianisation of the analogous formula for the case of the gg channel given in Ref. [119].

In Tab. 2, we show our results for different orders in the different partonic channels. Our predictions for the NLO EW corrections are in very good agreement with those of Ref. [55]. The small difference for the $q\bar{q}$ channel is related to the fact we are using a finite lepton mass. Nonetheless, such a difference is well below the 0.01% level for fiducial cross sections, confirming that the power corrections in the lepton mass are indeed negligible for the chosen value.

Turning to higher orders, we observe a convincing agreement for the NNLO QCD corrections in all considered partonic channels with the corresponding results of Ref. [55]. The choice of *product cuts* [108] in Eq. (13) alleviates the problem of linear power corrections in the q_T subtraction formalism, leading to a better convergence of the method. Concerning the mixed QCD–EW corrections, we find agreement within uncertainties in the all-quark channel, which contains the genuine two-loop virtual contribution, and in the gluon–photon channel. Our result for the correction in the quark–photon channel is about 10% larger than the corresponding result of Ref. [55]. We note that Ref. [55] quotes an overall 1% uncertainty on the mixed QCD–EW corrections, but does not provide the numerical accuracy of the separate partonic contributions, so we are unable to assess the significance of this small discrepancy. In the quark–gluon channel, however, the discrepancy is definitely significant, our result being about a factor of two larger than the corresponding result of Ref. [55]. Despite our efforts and two completely independent numerical implementations, we were unable to explain this difference. After extensive cross checks, the authors of Ref. [55] informed us that they found a bug in their implementation. After fixing this bug their result in the quark–gluon is in perfect agreement with ours. More detailed checks, also at the differential level, are needed for a thorough comparison of the two calculations.

In Fig. 7, we study the dependence on the slicing parameter r_{cut} for the nominal lepton mass $m_\ell = m_\mu$ and for another mass value, $m_\ell = 1 \text{ GeV}$. The results are consistent with the assumption that the residual dependence on the lepton mass, after the cancellation of large logarithms between real and virtual contributions, is indeed power suppressed. We observe that the power corrections of the lepton mass have a marginal impact on the final result. For comparison, the effect of increasing the lepton mass to $m_\ell = 1 \text{ GeV}$ on NLO EW corrections is

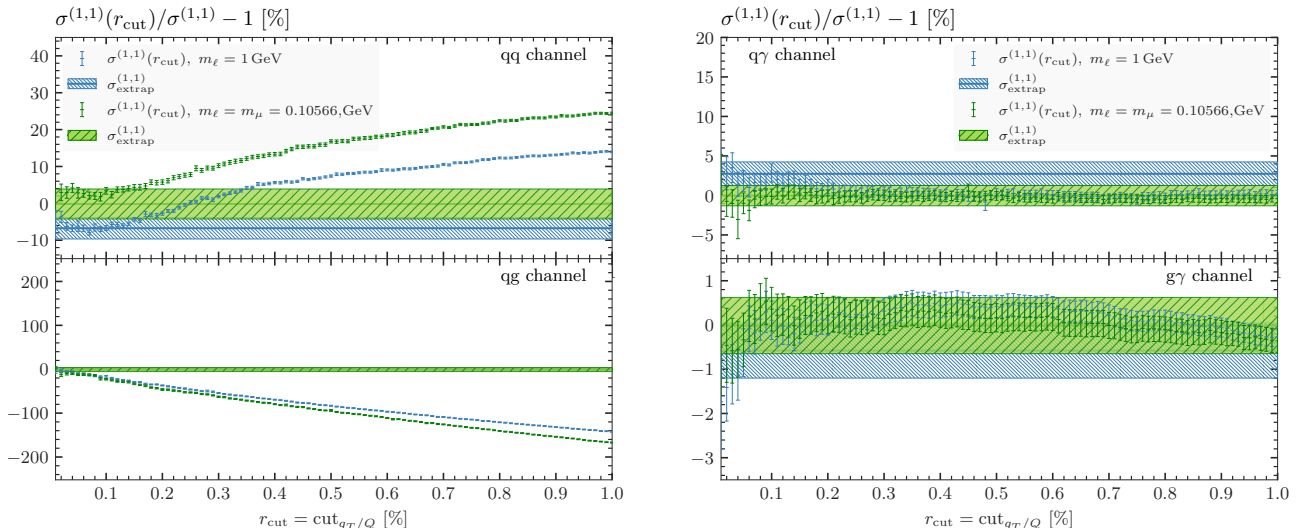


Figure 7: Mixed QCD–EW corrections $\sigma^{(1,1)}$ as functions of the parameter r_{cut} in the all-quark (top left), quark–gluon (bottom left), quark–photon (top right) and gluon–photon (bottom right) channels for two different values of the final-state lepton mass, $m_\ell = 1 \text{ GeV}$ and $m_\ell = m_\mu$. Our results are normalised to the $m_\ell = m_\mu$ result in the $r_{\text{cut}} \rightarrow 0$ limit.

to reduce them by about 3%. Our results provide a strong confirmation that the cancellation of all IR divergencies in our method does take place correctly, thereby providing a stringent check of the whole calculation.

5 Summary

In this paper we have reported on the complete computation of the mixed QCD–EW corrections to the neutral-current Drell–Yan process. Our calculation does not rely on any approximation and holds in the entire range of dilepton invariant masses. We have presented selected phenomenological results for several kinematical distributions in the case of bare muons both in the resonant and in the high invariant-mass region.

The impact of the newly computed corrections is typically at the few per mille level for the fiducial cross section, but can reach the percent level in some kinematical regions, and for specific kinematical distributions like the invariant mass of the dilepton pair for bare muons, thereby confirming their relevance for precision studies of the Drell–Yan process. The lepton-pair rapidity distribution receives mixed QCD–EW corrections ranging from few per mille in the central region up to 1% at large rapidities, with a possible impact on precision fits of the proton PDFs.

We have also considered the forward–backward asymmetry $A_{FB}(m_{\ell\ell})$, which is one of the main observables for the determination of the leptonic effective weak mixing angle at hadron colliders and is also potentially sensitive to new heavy resonances [75]. We studied the impact of the mixed QCD–EW corrections on $A_{FB}(m_{\ell\ell})$ by using the same setup adopted in the CMS measurement at $\sqrt{s} = 8 \text{ TeV}$ [116]. We have shown that the impact of mixed QCD–EW

corrections is significant. Since part of the effect is known to be due to QED final-state radiation and is therefore accounted for in the experimental analyses, a precise estimate of the impact of the computed corrections in $\sin^2 \theta_{\text{eff}}^\ell$ determinations will require more detailed studies.

We have finally applied our calculation to the case of dressed leptons. We have shown that in this case our numerical computation is sufficiently stable to be extrapolated to the massless limit. This is definitely non-trivial, given that the calculation is carried out by using a slicing method [88] in which final-state collinear singularities are regulated just with a finite lepton mass. We were able to compare our results for dressed leptons to those of Ref. [55]. We found a relatively good agreement between the corresponding results for the fiducial cross section except for the quark–gluon channel, for which our results disagree with those of Ref. [55]. This disagreement disappears after the correction of a bug in the implementation of Ref. [55]. Further cross-checks with the authors of Ref. [55] are ongoing.

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